## GENERAL APTITUDE

1. The search engine's business model $\qquad$ around the fulcrums of trust.
(A) Sinks
(B) Bursts
(C) Plays
(D) Revolves

## 01. Ans: (D)

Sol: Means to cause to go round or rotate.
02. Ten friends planned to share equally the cost of buying a gift for their teacher. When two of them decided not to contribute, each of the other friends had to pay Rs. 150 more. The cost of the gift was Rs $\qquad$
(A) 12000
(B) 6000
(C) 666
(D) 3000
02. Ans: (B)

Sol: Let the cost of gift = Rs. x
if 10 friends equally contribute, share per head $\frac{x}{10}$
if 8 friends equally contribute, share per head $\frac{x}{8}$ increase in share per head $=\frac{x}{8}-\frac{x}{10}=\frac{x}{40}$ given, $\frac{x}{40}=R s 150$
$\therefore \mathrm{X}=$ Rs 6000
03. Two cars start at the same time from the same location and go in the same direction. The speed of the first car is $50 \mathrm{~km} / \mathrm{h}$ and the speed of the second car is $60 \mathrm{~km} / \mathrm{h}$. The number of hours it takes for the distance between the two cars to be 20 km is $\qquad$
(A) 1
(B) 6
(C) 2
(D) 3

## 03. Ans: (C)

Sol: Relative speed of the two cars (same direction) $=60 \mathrm{~km} / \mathrm{hr}-50 \mathrm{~km} / \mathrm{hr}=10 \mathrm{~km} / \mathrm{hr}$ So, time taken $=\frac{20 \mathrm{~km}}{10 \mathrm{~km} / \mathrm{hr}}=2 \mathrm{hrs}$
04. The expenditure on the project $\qquad$ as follow: equipment Rs. 20 lakhs, salaries Rs. 12 laksh, and contingency Rs. 3 lakhs.
(A) breaks
(B) breaks down
(C) break Down
(D) break

## 04. Ans: (B)

Sol: Means to divide into parts to be analyzed.
05. A court is to a judge as $\qquad$ is to a teacher.
(A) a punishment
(B) a student
(C) a syllabus
(D) a school

## 05. Ans: (D)

Sol: Analogy type (person and work place) A judge decides in the courts just as a teacher teaches in the school. The obvious analogical relationship is a court: judge : : school : teacher.
06. "A recent High Court judgement has sought to dispel the idea of begging as a disease which leads to its stigmatization and criminalization - and to regard it as a symptom. The underlying disease is the failure of the state to protect citizens who fall through the social security net". Which one of the following statements can be inferred from the given passage?
(A) Beggars are created because of the lack of social welfare schemes
(B) Beggars are lazy people who beg because they are unwilling to work
(C) Begging has to be banned because it adversely affects the welfare of the state
(D) Begging is an offence that has to be dealt with firmly
06. Ans: (A)

Sol: The last sentence infers this (The underlying disease is the failure of the state to protect citizens who fall through the social security net").
07. In the given diagram, teachers are represented in the triangle, researchers in the circle and administrators in the rectangle. Out of the total number of the people, the percentage of administrators shall be in the range of $\qquad$ .
(A) 0 to 15
(B) 16 to 30
(C) 46 to 60
(D) 31 to 45

## Teachers


07. Ans: (D)

Sol: Total number of administrators $=10+20+20=50$
Total number of people $=70+10+20+20+40=160$
Thus, percentage of administrators $=\frac{50}{160} \times 100=31.25 \%$
i.e., it is range 31 to 45
08. Three of the five students allocated to a hostel put in special requests to the warden. Given the floor plant of the vacant rooms, select the allocation plan that will accommodate all their request. Request by X: Due to pollen allergy, I want to avoid a wing next to the garden.
Request by Y: I want to live as far from the washrooms as possible, since I am very sensitive to smell.

Request by Z: I believe in Vasstu and so want to stay in the South-west wing.
The shaded rooms are already occupied. WR is washroom.

(B)

(C)


08. Ans: (B)

Sol: Only option (B) satisfies the request of Z . it also satisfies the request of X and Y .
Hence, option (B) is correct.
09. In a college, there are three student clubs. Sixty students are only in the Drama club, 80 students are only in the Dance club, 30 students are only in the Maths club, 40 students are in both Drama and Dance clubs, 12 students are in both Dance and Maths club, 7 students are in both Drama and Maths clubs, and 2 students are in all the clubs. If $75 \%$ of the students in the college are not in any of these clubs, then the total number of students in the college is
(A) 1000
(B) 975
(C) 900
(D) 225

## 09. Ans: (C)

Sol: The total students is clubs can be found out using the relation of venn - diagram


As per question

$$
\begin{aligned}
& \text { only drama club }=60 \text { i.e., } a=60 \\
& \text { only dance club }=80 \text { i.e., } c=80 \\
& \text { only maths club }=30 \text { i.e., } g=30 \\
& \text { all time clubs }=2 \text { i.e., } e=2
\end{aligned}
$$

both Drama and Dance Clubs $=40$ i.e., $b+e=40 \Rightarrow b=40-2-=38$
both Dance and Maths Clubs $=12$ i.e., $e+f=12 \Rightarrow f=12-2=10$
both Drama and Maths clubs $=7$ i.e., $d+e=7 \Rightarrow d=7-2=5$
Thus, three students to who are in at least one of the clubs $=\mathrm{a}+\mathrm{b}+\mathrm{c}+\mathrm{d}+\mathrm{e}+\mathrm{f}+\mathrm{g}$

$$
=60+38+80+5+2+10+30=225
$$

This represents only $25 \%$ of students as remaining $75 \%$ are not in only of the clubs, So, $25 \%$ of total students $=225$

$$
\frac{1}{4} \times \text { Total students }=225
$$

$\therefore$ Total students $=900$
10. The police arrested four criminals $-\mathrm{P}, \mathrm{Q}, \mathrm{R}$ and S . The criminals knew each other. They made the following statements:

P says "Q committed the crime"
$Q$ says "S committed the crime"
R says "I did not do it."
S says "What Q said about me is false."
Assume only one of the arrested four committed the crime and only one of the statements made above is true. Who committed the crime?
(A) S
(B) P
(C) R
(D) Q
10. Ans: (C)

Sol: Assume that R committed the crime, the statements made by P is incorrect, Q is incorrect, R is incorrect but the statement made by $S$ will be correct.

Hence, option (C) is correct.

## COMPUTE SCIENCE AND INFORMATION TECHNOLOGY

1. Compute $\lim _{x \rightarrow 3} \frac{x^{4}-81}{2 x^{2}-5 x-3}$
(A) 1
(B) Limit does not exits
(C) $53 / 12$
(D) $108 / 7$
2. Ans: (D)

Sol: $\underset{x \rightarrow 3}{\operatorname{Lt}} \frac{x^{4}-81}{2 x^{2}-5 x-3}\left(\frac{0}{0}\right)$ In det er min ant form
$=\operatorname{Lt}_{\mathrm{x} \rightarrow 3} \frac{4 \mathrm{x}^{3}}{4 \mathrm{x}-5}(\mathrm{~L}-$ Hospital rule $)$
$=\frac{4 \times 27}{12-5}=\frac{108}{7}$
02. In 16-bit 2's complement representation, the decimal number -28 is
(A) 1000000011100100
(B) 0000000011100100
(C) 1111111111100100
(D) 1111111100011100
02. Ans: (C)

Sol:

| 2 | 28 |
| :--- | :---: |
|  | $14-0$ |
|  | $14-0$ |
| 2 | $3-1$ |
|  | $1-1$ |

$(28)_{10}=(11100)_{2}$
$+28=011100$
$\Downarrow 2$ 's complement
$-28=100100$

To represent in 16 bits, using sign extension place 1's at MSB for Negative numbers. $(-28)_{10}=\left[\begin{array}{lllll}1111 & 1111 & 1110 & 0100\end{array}\right]_{2}$
03. Consider a sequence of 14 elements: $\mathrm{A}=[-5,-10,6,3,-1,-2,13,4,-9,-1,4,12,-3,0]$.

The subsequence sum $S(i, j)=\sum_{k=1}^{j} A[k]$. Determine the maximum of $S(i, j)$ where $0 \leq \mathrm{i} \leq \mathrm{j}<14$. (Divide and conquer approach may be used).
03. Ans: 29
04. Consider the following C program:
\#include <stdio.h>
int jumble (int $x$, int $y$ )
\{
$x=2 * x+y ;$
return x ;
\}
int main( )
\{
int $\mathrm{x}=2, \mathrm{y}=5$;
$y=$ jumble $(y, x)$
$x=$ jumble ( $\mathrm{y}, \mathrm{x})$;
printf("\%d \n", x);
return 0 ;
\}
The value printed by the program is $\qquad$ .
04. Ans: 26

Sol:

$\left.\begin{array}{cc}x & y \\ \frac{5}{5} & \boxed{2} \\ x=2 * x+y\end{array} \right\rvert\, 12$ return $x$
05. Consider three concurrent processes P1, P2 and P3 as shown below, which access a shared variable D that has been initialized to 100 .

| P1 | P2 | P3 |
| :---: | :---: | :---: |
| $:$ | $:$ | $:$ |
| $:$ | $:$ | $:$ |
| $\mathrm{D}=\mathrm{D}+20$ | $\mathrm{D}=\mathrm{D}-50$ | $\mathrm{D}=\mathrm{D}+10$ |
| $:$ | $:$ | $:$ |
| $:$ | $:$ | $:$ |

The processes are executed on a uniprocessor system running a time-shared operating system. If the minimum and maximum possible values of D after the three process have completed execution are X and Y respectively, then the value of $\mathrm{Y}-\mathrm{X}$ is
05. Ans: 80

Sol: $\mathrm{D}=100$ initially


## Sequence of Execution:

for max value:
(I)
 $D=130=Y$
for min value: (A)
(B) I II III X Y

$D=50=X$

## $\therefore Y-X=130-50=80$

6. Consider $\mathrm{Z}=\mathrm{X}-\mathrm{Y}$, where $\mathrm{X}, \mathrm{Y}$ and Z are all in sign-magnitude form. X and Y are each represented in $n$ bits. To avoid overflow, the representation of Z would require a minimum of:
(A) $\mathrm{n}+1$ bits
(B) $\mathrm{n}-1$ bits
(C) $n+2$ bits
(D) $n$ bits
7. Ans: (A)

Sol: After adding two numbers of " n " bit data, the longest result size is " $\mathrm{n}+1$ " bits
Example: Let $\mathrm{X}=+3 ; \quad \mathrm{Y}=-2$
Representing above in sign magnitude form
$X=+3=011[n=3$ bits required]
$\mathrm{Y}=-2=110[\mathrm{n}=3$ bits required]
Now, $X-Y=3-(-2)=+5=0101(\mathrm{n}=4=3+1$ bits required]
07. The chip select logic for a certain DRAM chip in a memory system design is shown below. Assume that the memory system has 16 address lines denoted by $\mathrm{A}_{15}$ to $\mathrm{A}_{0}$. What is the range of addresses (in hexadecimal) of the memory system that can get enabled by the chip select (CS) signal?

(A) C800 to CFFF
(B) C800 to C8FF
(C) DA00 to DFFF
(D) CA00 to CAFF

## 07. Ans: (A)

Sol: Based on given system design $\mathrm{A}_{13}$ and $\mathrm{A}_{12}$ should be 0 (zero) ; and $\mathrm{A}_{15} \mathrm{~A}_{14}$ and $\mathrm{A}_{11}$ should be ' 1 ' (one); to enable chip select (CS).

Hence address range will differ for address bits $\mathrm{A}_{10}$ to $\mathrm{A}_{0}$.

08. Let G be an arbitrary group. Consider the following relations on G :
$\mathbf{R}_{1}: \forall \mathrm{a}, \mathrm{b} \in \mathrm{G}, \mathrm{a} \mathrm{R}_{1} \mathrm{~b}$ if any only if $\exists \mathrm{g} \in \mathrm{G}$ such that $\mathrm{a}=\mathrm{g}^{-1} \mathrm{bg}$
$\mathbf{R}_{2}: \forall \mathrm{a}, \mathrm{b} \in \mathrm{G}, \mathrm{a} \mathrm{R}_{2} \mathrm{~b}$ if and only if $\mathrm{a}=\mathrm{b}^{-1}$
Which of the above is/are equivalence relation/relations?
(A) $\mathrm{R}_{1}$ only
(B) Neither $\mathrm{R}_{1}$ nor $\mathrm{R}_{2}$
(C) $\mathrm{R}_{1}$ and $\mathrm{R}_{2}$
(D) $\mathrm{R}_{2}$ only
08. Ans: (A)

Sol: Let $G$ be an arbitrary group consider two relations $\mathrm{R}_{1}$ and $\mathrm{R}_{2}$
$\mathrm{R}_{1}: \forall \mathrm{a}, \mathrm{b} \in \mathrm{G}, \mathrm{a} \mathrm{R}_{1} \mathrm{~b}$ iff $\exists \mathrm{g} \in \mathrm{G}$ such that $\mathrm{a}=\mathrm{g}^{-1} \mathrm{bg}$
$\mathrm{R}_{2}: \forall \mathrm{a}, \mathrm{b} \in \mathrm{G}, \mathrm{a} \mathrm{R}_{2} \mathrm{~b}$ iff $\mathrm{a}=\mathrm{b}^{-1}$, which of these $\mathrm{R}_{1}$ and $\mathrm{R}_{2}$ are equivalence relations.
$\mathrm{R}_{1}$ is reflexive $\mathrm{a}=\mathrm{e}^{-1}$ ae $\forall \mathrm{a} \in \mathrm{G}, \mathrm{e} \in \mathrm{G}$.
$R_{1}$ is symmetric
$a R_{1} b \Rightarrow a=g^{-1} b g$ for some $g \in G$

$$
\begin{gathered}
\Rightarrow \mathrm{gag}^{-1}=\mathrm{b} \\
\Rightarrow\left(\mathrm{~g}^{-1}\right)^{-1} \mathrm{ag}^{-1}=\mathrm{b} \Rightarrow \mathrm{~b} \mathrm{R}_{1} \mathrm{a}
\end{gathered}
$$

$R_{1}$ is transitive
Let $\mathrm{R}_{1} \mathrm{~b}, \mathrm{~b} \mathrm{R}_{1} \mathrm{c}$

$$
\begin{aligned}
& \mathrm{a}=\mathrm{g}_{1}^{-1} \mathrm{bg}_{1}, \mathrm{~b}=\mathrm{g}_{2}^{-1} \mathrm{cg}_{2} \\
& \Rightarrow \mathrm{a}=\mathrm{g}_{1}^{-1}\left(\mathrm{~g}_{2}^{-1} \mathrm{cg}_{2}\right) \mathrm{g}_{2} \\
& =\left(\mathrm{g}_{1}^{-1} \cdot \mathrm{~g}_{2}^{-1}\right) \mathrm{c}\left(\mathrm{~g}_{2} \mathrm{~g}_{1}\right) \Rightarrow \mathrm{a}=\left(\mathrm{g}_{2} \mathrm{~g}_{1}\right)^{-1} \mathrm{c}\left(\mathrm{~g}_{2} \mathrm{~g}_{1}\right)
\end{aligned}
$$

$\Rightarrow R_{1}$ is transitive and $R_{1}$ is equivalence relation.
$R_{2}$ is not reflexive since $\forall a \in G a=a^{-1}$ need not be true.
$\therefore \mathrm{R}_{1}$ is equivalence relation but not $\mathrm{R}_{2}$
So, option (A) is correct.
09. If L is a regular language over $\Sigma=\{\mathrm{a}, \mathrm{b}\}$, which one of the following languages is NOT regular?
(A) $L \cdot L^{R}=\left\{x y \mid x \in L, y^{R} \in L\right\}$
(B) $\operatorname{Prefix}(L)=\left\{x \in \Sigma^{*} \mid \exists y \in \Sigma^{*}\right.$ such that $\left.x y \in L\right\}$
(C) $\left\{w w^{R} \mid w \in L\right\}$
(D) Suffix (L) $=\left\{y \in \Sigma^{*} \mid \exists x \in \Sigma^{*}\right.$ such that $\left.x y \in L\right\}$
09. Ans: (C)

Sol: A. $L$ is regular $\Rightarrow L^{R}$ is regular
$L . L^{R}$ is also regular.
B. If $L$ is regular then prefix $(\mathrm{L})$ is also regular
C. $w w^{R}$ is possible as not regular
D. If $L$ is regular then suffix $(\mathrm{L})$ is also regular.

Note: If $L$ is finite language then option (C) can be regular.
10. Which of the following protocol pairs can be used to send and retrieye e-mails (in that order)?
(A) IMAP, SMTP
(B) SMTP, POP3
(C) IMAP, POP3
(D) SMTP, MIME
10. Ans: (B)

Sol: SMTP: Simple Mail Transfer Protocol used to send mail
POP 3: Post Office Protocol used to retrieve mail
11. A certain processor uses a fully associative cache of size 16 kB . The cache block size is 16 bytes. Assume that the main memory is byte addressable and uses a 32 -bit address. How many bits are required for the Tag and the Index fields respectively in the addresses generated by the processor?
(A) 28 bits and 4 bits
(B) 24 bits and 4 bits
(C) 24 bits and 0 bits
(D) 28 bits and 0 bits
11. Ans: (D)

Sol: Cache size $=16 \mathrm{~KB}$
Block Size $=16 \mathrm{~B}=2^{4} \mathrm{~B} \Rightarrow$ Block offset $=4-$ bits
Main memory address $=32-$ bits
In fully associative cache main memory address format

Main Memory address

$32-(4)=28-$ bits
In fully associative cache, there is no any index.
Hence,
Tag $=28$ bits
Index $=0$ - bits
12. Which one of the following kinds of derivation is used by LR parsers?
(A) Rightmost
(B) Leftmost in reverse
(C) Rightmost in reverse
(D) Leftmost
12. Ans: (C)

Sol: LR parser is bottom up passer.
Every bottom up parser uses "Reverse of RMD".
So, RMD in reverse is correct.
13. Consider the following C program:

```
#include <stdio.h>
int main()
{
    int arr[ ] ={1,2,3,4,5,6,7, 8, 9, 0, 1, 2, 5}, *ip = arr + 4;
    printf("%d\n", ip[1]);
    return 0;
}
```

The number that will be displayed on execution of the program is $\qquad$ .
13. Ans: 6

Sol:

$=100+(4 \times 2)$
$=108 / \mathrm{ip}$ pointer pointing to 108
printf("\%d", ip[1]);
output is 6
14. Which one of the following is NOT a valid identity?
(A) $(\mathrm{x} \oplus \mathrm{y}) \oplus \mathrm{z}=\mathrm{x} \oplus(\mathrm{y} \oplus \mathrm{z})$
(B) $x \oplus y=x+y$, if $x y=0$
(C) $(\mathrm{x}+\mathrm{y}) \oplus \mathrm{z}=\mathrm{x} \oplus(\mathrm{y}+\mathrm{z})$
(D) $x \oplus y=\left(x y+x^{\prime} y^{\prime}\right)^{\prime}$
14. Ans: (C)

Sol: (A). L.H.S

$$
\begin{aligned}
& {[\mathrm{x} \oplus \mathrm{y}] \oplus \mathrm{z} } \\
= & {[\mathrm{x} \overline{\mathrm{y}}+\overline{\mathrm{x}} \mathrm{z}] \oplus \mathrm{z} } \\
= & \mathrm{x} \overline{\mathrm{y}} \overline{\mathrm{z}}+\overline{\mathrm{x}} \mathrm{y} \overline{\mathrm{z}}+\overline{\mathrm{x}} \overline{\mathrm{y}} \mathrm{z}+\mathrm{xyz}
\end{aligned}
$$

## R.H.S

```
x}\oplus[y\oplusz
    =x }\oplus[y\overline{z}+\overline{y}z]=x\overline{y}\overline{z}+\overline{x}y\overline{z}+\overline{x}\overline{y}z+xy
```

L.H.S $==$ R.H.S hence, option (A) correct.
(B). $x \oplus y=\overline{[x \odot y]}$
$=[\overline{\mathrm{x}} \overline{\mathrm{y}}+x y]$
$\Rightarrow \overline{\bar{x} \bar{y}+0}$ [If $x y=0]$

$$
\Rightarrow \overline{\bar{x}}+\overline{\bar{y}}
$$

$$
\Rightarrow(x+y)
$$

So, $2^{\text {nd }}$ statement is also correct
(C). L.H.S $(x+y) \oplus z=(x+y) \cdot \bar{z}+\overline{(x+y)} \cdot z$
R.H.S $x \oplus(y+z)=x \overline{(y+z)}+\bar{x}(y+z)$

Here L.H.S $\neq$ R.H.S
hence, option (C) is not a valid statement.
(D) $\cdot[x \oplus y]=\overline{[x \odot y]}=\overline{[x y+\bar{x} \bar{y}]}$

So, option (D) is also correct
15. For $\Sigma=\{a, b\}$, let us consider the regular language $L=\left\{x \mid x=a^{2+3 k}\right.$ or $\left.x=b^{10+12 k}, k \geq 0\right\}$. Which one of the following can be a pumping length (the constant guaranteed by the pumping lemma) for L ?
(A) 9
(B) 24
(C) 5
(D) 3
15. Ans: (C)

Sol: $L=\left\{x \mid x=a^{2+3 k}\right.$ or $\left.x=b^{10+12 k}\right\}$
Let pumping length $=5$
So, $|w| \geq 5$
and $\mathrm{w}=\mathrm{xyz}$ where $\mathrm{x}=\mathrm{aa}, \mathrm{y}=\mathrm{aaa}, \mathrm{z}=\varepsilon$
such that $|x y| \leq 5,|y| \neq 0$
$\forall \mathrm{i}, \mathrm{xy}{ }^{\mathrm{i}} \mathrm{z} \in \mathrm{L}$
i.e., aa(aaa) ${ }^{i} \in L$

So, option (C) is correct.
16. Consider the following two statements about database transaction schedules:
I. Strict two-phase locking protocol generates conflict serializable schedules that are also recoverable.
II. Timestamp-ordering concurrency control protocol with Thomas Write Rule can generate view serializable schedules that are not conflict serializable.

Which of the above statements is/are TRUE?
(A) Both I and II
(B) I only
(C) II only
(D) Neither I nor II
16. Ans: (A)

Sol: I. Strict 2 PL requires that all the executive mode locks taken by the transactions must be held until it commits, so the strict 2PL schedules are cascadeless and also recoverable.
II. Time stamp ordering protocols can generate conflict serializable schedules but time stamp ordering protocol with Thomas write rule can generate view serializable schedules.
17. Which one of the following statements is NOT correct about the $\mathrm{B}^{+}$tree data structure used for creating an index of a relational database table?
(A) $\mathrm{B}^{+}$tree is a height-balanced tree
(B) Non-leaf nodes have pointers to data records
(C) Each leaf node has a pointer to the next leaf node
(D) Key values in each node are kept in sorted order
17. Ans: B

Sol: - $\mathrm{B}^{+}$Tree is a height balanced search tree

- non leaf nodes have pointers to the next level nodes but not to the data records
- All the leaf nodes are connected with a pointer $\mathbf{P}_{\text {next }}$
- All the key values in each node are kept in sorted order.

18. The value of $3^{51} \bmod 5$ is $\qquad$ .
19. Ans: 2

Sol: Fast exponential modular arithmetic (used in RSA)
$3^{51} \bmod 5 \Rightarrow a^{e} \bmod 5$

$$
51=\text { Binary } \begin{array}{|l|l|l|l|l|l|}
\hline 1 & 1 & 0 & 0 & 1 & 1 \\
\hline 1 & 4 & 4 & 1 & 3 & 4 \\
3 & 2 & x & x & 4 & (2) \\
\hline
\end{array}
$$

19. Consider the grammar given below:
$\mathrm{S} \rightarrow \mathrm{Aa}$
$\mathrm{A} \rightarrow \mathrm{BD}$
$\mathrm{B} \rightarrow \mathrm{b} \mid \in$
$\mathrm{D} \rightarrow \mathrm{d} \mid \in$
Let $\mathrm{a}, \mathrm{b}, \mathrm{d}$ and $\$$ be indexed as follows:

| a | b | d | $\$$ |
| :---: | :---: | :---: | :---: |
| 3 | 2 | 1 | 0 |

Compute the FOLLOW set of the non-terminal B and write the index values for the symbols in the FOLLOW set in the descending order. (For example, if the FOLLOW set is $\{\mathrm{a}, \mathrm{b}, \mathrm{d}, \$\}$, then the answer should be 3210)
19. Ans: 31

Sol: $\mathrm{S} \rightarrow \mathrm{Aa}$
$\mathrm{A} \rightarrow \mathrm{BD}$
$\mathrm{B} \rightarrow \mathrm{b} \mid \varepsilon$
$\mathrm{D} \rightarrow \mathrm{d} \mid \varepsilon$
Follow $(B)=\{d\} \cup$ Follow $(A)$

$$
=\{\mathrm{d}\} \cup\{\mathrm{a}\}=\{\mathrm{a}, \mathrm{~d}\}
$$

Descending order: 31 where $\mathrm{a}=3$ and $\mathrm{d}=1$
20. Let X be a square matrix. Consider the following two statements on X .
I. X is invertible.
II. Determinant of X is non-zero.

Which one of the following is TRUE?
(A) I implies II; II does not imply I
(B) II implies I; I does not imply II
(C) I does not imply II; II does not imply I
(D) I and II are equivalent statements
20. Ans: (D)

Sol: Given we know that a matrix invertiable $\Leftrightarrow \operatorname{det}$ is non zero.
Statement $\mathrm{I}: \mathrm{X}$ is invertiable, ( X is a square matrix)
Statement II: Determinant of X is non zero.
$\therefore$ Both statements are equivalent
$\therefore$ The absolute value of product of eigen value
21. The following C program is executed on a Unix/Linux system: \#include <unistd. h>
int main()
\{ int i;

$$
\text { for }(\mathrm{i}=0 ; \mathrm{i}<10 ; \mathrm{i}++)
$$

if (i \% $2==0$ ) fork ( );
return 0 ;
\}
The total number of child processes created is $\qquad$

22. Let $U=\{1,2, \ldots, n\}$. Let $A=\{(x, X) \mid x \in X, X \subseteq U\}$. Consider the following two statements on |A|.
I. $|\mathrm{A}|=\mathrm{n} 2^{\mathrm{n}-1}$
II. $|A|=\sum_{k=1}^{n} k\binom{\mathrm{n}}{\mathrm{k}}$

Which of the above statements is/are TRUE?
(A) Neither I nor II
(B) Only II
(C) Both I and II
(D) Only I
22. Ans: (C)

Sol: $U=\{1,2, \ldots \ldots . . n\}$
$A=\{(x, X) \mid x \in X$ and $X \subseteq U\}$
Number of non empty subsets of $\mathrm{X}=\mathrm{C}(\mathrm{n}, 1)+\mathrm{C}(\mathrm{n}, 2)+\ldots \ldots .+\mathrm{C}(\mathrm{n}, \mathrm{n})$
Number of elements in $\mathrm{A}=\sum_{k=1}^{n} k C(n, k)$
Using Binomial Theorem, we have $\sum_{k=1}^{n} k C(n, k)=$ n. $2^{\mathrm{n}-1}$
$\therefore$ Both I and II are true.
23. Let $G$ be an undirected complete graph on $n$ vertices, where $n>2$. Then, the number of different Hamiltonian cycles in $G$ is equal to
(A) 1
(B) $\frac{(n-1)!}{2}$
(C) n !
(D) $(\mathrm{n}-1)$ !
23. Ans: (B)

Sol: Number of Hamilton cycles in a complete graph of $n$ vertices $(n>2)$ is $\frac{(n-1)!}{2}$ Option (B) is correct.
24. An array of 25 distinct elements is to be sorted using quicksort. Assume that the pivot element is chosen uniformly at random. The probability that the pivot element gets placed in the worst possible location in the first round of partitioning (rounded off to 2 decimal places) is $\qquad$
24. Ans: 0.08

Sol: Probability of placing pivot element in worst possible location $=\frac{1}{25}+\frac{1}{25}$

$$
=\frac{2}{25}=0.08
$$

25. Two numbers are chosen independently and uniformly at random from the set $\{1,2, \ldots, 13\}$. The probability (rounded off to 3 decimal places) that their 4 bit (unsigned) binary representations have the same most significant bit is $\qquad$ .

## 25. Ans: 0.461

Sol: In 4 -digit binary numbers, we need to represent 1 to 13 . There are 7 binary numbers with most significant bit ' 0 ' and 6 numbers with most significant bit ' 1 '.
Required probability $=\frac{{ }^{6} \mathrm{C}_{2}}{{ }^{13} \mathrm{C}_{2}}+\frac{{ }^{7} \mathrm{C}_{2}}{{ }^{13} \mathrm{C}_{2}}$

$$
\begin{aligned}
& =\frac{6 \times 5}{13 \times 12}+\frac{7 \times 6}{13 \times 12} \\
& =\frac{5}{26}+\frac{7}{26}=\frac{12}{26}=0.461
\end{aligned}
$$

26. Consider three 4 variable functions $f_{1}, f_{2}$ and $f_{3}$, which are expressed in sum of midterms as $\mathrm{f}_{1}=\Sigma(0,2,5,8,14)$
$\mathrm{f}_{2}=\Sigma(2,3,6,8,14,15)$
$\mathrm{f}_{3}=\Sigma(2,7,11,14)$
For the following circuit with one AND gate and one XOR gate, the output function $f$ can be expressed as:

(A) $\Sigma(2,14)$
(B) $\Sigma(2,7,8,11,14)$
(C) $\Sigma(0,2,3,5,6,7,8,11,14,15)$
(D) $\Sigma(7,8,11)$
27. Ans: (D)

Sol: $\mathrm{f}=\left[\mathrm{f}_{1} \cdot \mathrm{f}_{2}\right] \oplus \mathrm{f}_{3}=\left[\mathrm{f}_{1} \cap \mathrm{f}_{2}\right] \oplus \mathrm{f}_{3}$
$\mathrm{f}_{1}=\Sigma(0,2,5,8,14)$
$\mathrm{f}_{2}=\Sigma(2,3,6,8,14,15)$
$\mathrm{f}_{1} . \mathrm{f}_{2}=\mathrm{f}_{1} \cap \mathrm{f}_{2}=\Sigma[2,8,14]\left\{\because\right.$ Common terms in $\mathrm{f}_{1}$ and $\left.\mathrm{f}_{2}\right\}$

| $\mathrm{f}_{1} \mathrm{f}_{2}$ | $\mathrm{f}_{3}$ | $\mathrm{f}_{1} \mathrm{f}_{2} \oplus \mathrm{f}_{3}$ |
| :---: | :---: | :---: |
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

[If both cases present then that term absent in f]
Now $f_{1} . \mathrm{f}_{2} \oplus \mathrm{f}_{3}$
$\mathrm{f}_{1} . \mathrm{f}_{2}=\Sigma(2,8,14)$
$\mathrm{f}_{3}=\Sigma(2,7,11,14)$
$\mathrm{f}=\mathrm{f}_{1} . \mathrm{f}_{2} \oplus \mathrm{f}_{3}=\Sigma(7,8,11)$
$\mathrm{f}=\Sigma(7,8,11)$
27. Suppose that in a IP over Ethernet network, a machine $X$ wishes to find the MAC address of another machine Y in its subnet. Which one of the following techniques can be used for this?
(A) X sends and ARP request packet to the local gateway's IP address which then finds the MAC address of $Y$ and sends to X .
(B) X sends an ARP request packet with broadcast IP address in its local subnet.
(C) X sends an ARP request packet to the local gateway's MAC address which then finds the MAC address of Y and sends to X .
(D) X sends an ARP request packet with broadcast MAC address in its local subnet.
27. Ans: (D)

Sol: X sends an ARP request packet with broadcast MAC address in its local subnet.


ARP request packet (Broadcast by X)
Source IP Address = X IP Address
Destination IP Address = Y IP Address
Source MAC Address $=\mathrm{X}$ MAC Address
Destination MAC Address $=$ BroadCast MAC Address
(FF : $: \mathrm{FF}:: \mathrm{FF}:: \mathrm{FF}:: \mathrm{FF} \because: \mathrm{FF}$ )
ARP Reply packet (Unicast by Y)
Source IP Address = Y IP Address
Destination IP Address $=X$ IP Address
Source MAC Address $=Y$ MAC Address
Destination MAC Address $=\mathbf{X}$ MAC Address
28. Consider the following grammar and the semantic actions to support the inherited type declaration attributes. Let $X_{1}, X_{2}, X_{3}, X_{4}, X_{5}$ and $X_{6}$ be the placeholders for the non-terminals $D, T, L$ or $L_{1}$ in the following table:

| Production rule | Semantic action |
| :--- | :--- |
| $\mathrm{D} \rightarrow \mathrm{TL}$ | $\mathrm{X}_{1}$.type $=\mathrm{X}_{2}$.type |
| $\mathrm{T} \rightarrow$ int | T.type $=$ int |
| $\mathrm{T} \rightarrow$ float | T.type $=$ float |
| $\mathrm{L} \rightarrow \mathrm{L}_{1}$, id | $\mathrm{X}_{3}$.type $=\mathrm{X}_{4}$ type <br> addType $\left(\right.$ id.entry, $\mathrm{X}_{5}$.type $)$ |
| $\mathrm{L} \rightarrow$ id | addType(id.entry, $\mathrm{X}_{6}$.type $)$ |

Which one of the following are the appropriate choices for $X_{1}, X_{2}, X_{3}$ and $X_{4}$ ?
(A) $\mathrm{X}_{1}=\mathrm{T}, \mathrm{X}_{2}=\mathrm{L}, \mathrm{X}_{3}=\mathrm{T}, \mathrm{X}_{4}=\mathrm{L}_{1}$
(B) $\mathrm{X}_{1}=\mathrm{T}, \mathrm{X}_{2}=\mathrm{L}, \mathrm{X}_{3}=\mathrm{L}_{1}, \mathrm{X}_{4}=\mathrm{T}$
(C) $\mathrm{X}_{1}=\mathrm{L}, \mathrm{X}_{2}=\mathrm{T}, \mathrm{X}_{3}=\mathrm{L}_{1}, \mathrm{X}_{4}=\mathrm{L}$
(D) $\mathrm{X}_{1}=\mathrm{L}, \mathrm{X}_{2}=\mathrm{L}, \mathrm{X}_{3}=\mathrm{L}_{1}, \mathrm{X}_{4}=\mathrm{T}$
28. Ans: (C)

Sol: $\mathrm{D} \rightarrow \mathrm{TL}\left\{\mathrm{X}_{1}\right.$.type $=\mathrm{X}_{2}$.type $\}$
$\mathrm{T} \rightarrow$ int $\{$ T.type $=$ int $\}$
$\mathrm{T} \rightarrow$ float $\{$ T.type $=$ float $\}$
$\mathrm{L} \rightarrow \mathrm{L}_{1}$, id $\left\{\mathrm{X}_{3}\right.$.type $=\mathrm{X}_{4}$.type, addType (id.entry, $\mathrm{X}_{5}$. type $\left.)\right\}$

L $\rightarrow$ id \{addType (id.entry, $\mathrm{X}_{6}$.type) $\}$
Case (i):
D $\rightarrow$ TL
$\mathrm{T} \rightarrow$ int
$\mathrm{T} \rightarrow$ float
Here, T.type is known as either int or float.
$\mathrm{D} \rightarrow$ TL semantic should assign T.type to L.type.
So. L.Type = T.type
Therefore, $\mathrm{X}_{1}=\mathrm{L}$ and $\mathrm{X}_{2}=\mathrm{T}$.
So, SDT is given below.
$\mathrm{D} \rightarrow \mathrm{TL}\{$ L.type $=$ T.type $\}$
$\mathrm{T} \rightarrow$ int $\{$ T.type $=$ int $\}$
$\mathrm{T} \rightarrow$ float $\{\mathrm{T}$. type $=$ float $\}$
$\mathrm{L} \rightarrow \mathrm{L}_{1}, \operatorname{id}\left\{\mathrm{~L}_{1}\right.$. type $=$ L.type; addType(id.entry, L.type) $\}$
$\mathrm{L} \rightarrow$ id \{addType(id.entry, L.type) \}
$X_{1}$ is $L, X_{2}$ is $T, X_{3}$ is $L_{1}, X_{4}$ is $L$
29. Consider the following statements:
I. The smallest element in a max-heap is always at a leaf node.
II. The second largest element in a max-heap is always a child of the root node.
III. A max-heap can be constructed from a binary search tree in $\Theta(\mathrm{n})$ time.
IV. A binary search tree can be constructed form a max-heap in $\Theta(\mathrm{n})$ time.

Which of the above statements are TRUE?
(A) I, III and IV
(B) II, III and IV
(C) I, II and III
(D) I, II and IV
29. Ans: (C)
30. What is the minimum number of 2 input NOR gates required to implement a 4 variable function expressed in sum of minterms form as $f=\Sigma(0,2,5,7,8,10,13,15)$ ?

Assume that all the inputs and their complements are available. Answer: $\qquad$ .
30. Ans: 3

Sol: Given $\mathrm{f}=\Sigma \mathrm{m}[0,2,5,7,8,10,13,15]$
To design with NOR gate function should be in POS form.

$$
\mathrm{f}_{\text {spos }}=\Pi \mathrm{M}[1,3,4,6,9,11,12,14]
$$


$\mathrm{f}=(\overline{\mathrm{B}}+\mathrm{D})(\mathrm{B}+\overline{\mathrm{D}})$ Now re-arrange
$\mathrm{f}=\overline{\overline{\mathrm{f}}}=\overline{\overline{(\mathrm{B}}+\mathrm{D})(\mathrm{B}+\overline{\mathrm{D}})}$
$\mathrm{f}=\overline{\overline{(\overline{\mathrm{B}}+\mathrm{D})}+\overline{(\mathrm{B}+\overline{\mathrm{D}})}}$
Given complement variables also available


Three NOR Gates are required
31. Consider the augmented grammar given below:

$$
\begin{aligned}
& \mathrm{S}^{\prime} \rightarrow \mathrm{S} \\
& \mathrm{~S} \rightarrow\langle\mathrm{~L}\rangle \mid \mathrm{id} \\
& \mathrm{~L} \rightarrow \mathrm{~L}, \mathrm{~S} \mid \mathrm{S}
\end{aligned}
$$

Let $\mathrm{I}_{0}=\operatorname{CLOSURE}\left(\left\{\left[\mathrm{S}^{\prime} \rightarrow \bullet \mathrm{S}\right]\right\}\right)$. The number of items in the set GOTO $\left(\mathrm{I}_{0},<\right)$ is $\qquad$ .

## 31. Ans: 5

Sol: $\mathrm{I}_{0}=$ Closure $\left(\mathrm{S}^{1} \rightarrow . \mathrm{S}\right)$

| $\mathrm{S}^{1} \rightarrow . \mathrm{S}$ <br> $\mathrm{S} \rightarrow .(\mathrm{L})$ <br> $\mathrm{S} \rightarrow . \mathrm{id}$ |
| :--- |
| $\mathrm{I}_{0}$ |

$\operatorname{goto}\left(\mathrm{I}_{0},<\right)$ contain 5 items
32. Let $\Sigma$ be the set of all bijections from $\{1, \ldots, 5\}$ to $\{1, \ldots 5\}$, where $i d$ denotes the identity function, i.e. $\mathrm{id}(\mathrm{j})=\mathrm{j}, \forall \mathrm{j}$. Let ${ }^{\circ}$ denote composition on functions. For a string $\mathrm{x}=\mathrm{x}_{1} \mathrm{x}_{2} \ldots \mathrm{x}_{\mathrm{n}} \in \Sigma^{\mathrm{n}}, \mathrm{n} \geq 0$, let $\pi(x)=x_{1}{ }^{\circ} \mathrm{x}_{2}{ }^{\circ}$ $\qquad$ $\mathrm{X}_{\mathrm{n}}$.

Consider the language $L=\left\{x \in \Sigma^{*} \mid \pi(x)=i d\right\}$. The minimum number of states in any DFA accepting L is $\qquad$ .
32. Ans: 120

Sol: Let $\sum$ be set of all bijections from $\{1,2\}$ to $\{1,2\}$
$\sum=\left\{\mathrm{x}_{1}, \mathrm{x}_{2}\right\}$ where $\mathrm{x}_{1}$ and $\mathrm{x}_{2}$ are two bijections.


2
$\mathrm{L}:=\left\{\mathrm{x} \in \sum^{\mathrm{n}} \mid \pi(\mathrm{x})=\mathrm{id}\right\}$

$$
\begin{aligned}
& \mathrm{x}_{1} \mathrm{o} \mathrm{x}_{1}=\mathrm{x}_{1} \\
& \mathrm{x}_{1 \mathrm{o}} \mathrm{x}_{2}=\mathrm{x}_{2} \\
& \mathrm{x}_{2} \mathrm{x}_{2}=\mathrm{x}_{1} \\
& \mathrm{x}_{2} \mathrm{o}_{1}=\mathrm{x}_{2}
\end{aligned}
$$

DFA that accepts L:


2 states are enough to accept L .
If we have $\sum$ that contain set of bijections from $\{1,2, \ldots . .5\}$ to $\{1,2, \ldots . . .5\}$ then $|\Sigma|=120$
$\Sigma=\left\{\mathrm{x}_{1}, \mathrm{x}_{2}, \mathrm{x}_{3}, \ldots \ldots \ldots, \mathrm{x}_{120}\right\}$.
Similarly, we can design DFA with 120 states.
33. Which one of the following languages over $\Sigma=\{\mathrm{a}, \mathrm{b}\}$ is NOT context-free?
(A) $\left\{w w^{R} \mid w \in\{a, b\}^{*}\right\}$
(B) $\left\{a^{n} b^{i} \mid i \in\{n, 3 n, 5 n\}, n \geq 0\right\}$
(C) $\left\{w^{n} b^{n} w^{R} \mid w \in\{a, b\}^{*}, n \geq 0\right\}$
(D) $\left\{w^{n} w^{R} b^{n} \mid w \in\{a, b\}^{*}, n \geq 0\right\}$
33. Ans: (D)

Sol: (A) $\left\{w w^{R} \mid w \in(a+b)^{*}\right\}$ is CFL
(B) $\left\{a^{n} b^{i} \mid i \in\{n, 3 n, 5 n\}, n \geq 0\right\}=\left\{a^{n} b^{n}\right\} \cup\left\{a^{n} b^{3 n}\right\} \cup\left\{a^{n} b^{5 n}\right\}$ is also CFL.
(C) $\left\{\mathrm{wa}^{\mathrm{n}} \mathrm{b}^{\mathrm{n}} \mathrm{w}^{\mathrm{R}} \mid \mathrm{w} \in(\mathrm{a}+\mathrm{b})^{*}, \mathrm{n} \geq 0\right\}$ is CFL

Equivalent CFG:

$$
\begin{aligned}
& \mathrm{S} \rightarrow \mathrm{aSa}|\mathrm{bSb}| \mathrm{A} \\
& \mathrm{~A} \rightarrow \mathrm{aAb} \mid \varepsilon
\end{aligned}
$$

(D) $\left\{w a^{n} w^{R} b^{n} \mid w \in(a+b)^{*}, n \geq 0\right\}$ is not CFL
34. The index node (inode) of a Unix-like file system has 12 direct, one single-indirect and one double-indirect pointers. The disk block size is 4 kB , and the disk block address is 32 -bits long. The maximum possible file size is (rounded off to 1 decimal place) $\qquad$ GB.
34. Ans: 4

Sol:
$\therefore$ Max. File Size $=48+4096+4194304$ KB

$$
=4198448 \mathrm{~KB}=4.0 \mathrm{~GB}
$$

35. Consider three machines M, N, and P with IP addresses 100.10.5.2, 100.10.5.5, and 100.10.5.6 respectively. The subnet mask is set to 255.255 .255 .252 for all the three machines. Which one of the following is true?
(A) $\mathrm{M}, \mathrm{N}$, and P belong to three different subnets
(B) Only N and P belong to the same subnet
(C) Only M and N belong to the same subnet
(D) M, N, and P all belong to the same subnet
36. Ans: (B)

Sol: Subnet Mask $\Rightarrow \underline{255.255 .255 .111111} 00$

## 2 bit HOST ID

$M \Rightarrow \underline{100 \cdot 10 \cdot 5 \cdot 000000} \underline{10}$
$\mathrm{N} \Rightarrow$ 100.10.5.000001 01
$\mathrm{P} \Rightarrow \underline{100 \cdot 10 \cdot 5 \cdot 000001} \underline{10}$
Only N \& P belongs to the same subnet because both have same fixed part.
36. Consider the following $C$ function.

```
void convert (int n)
{
            if (n<0)
                printf ("%d", n);
            else
            {
                convert (n/2);
                printf ("%d", n%2);
            }
                            }
```

Which one of the following will happen when the function convert is called with any positive integer n as argument?
(A) It will print the binary representation of n and terminate
(B) It will print the binary representation of n in the reverse order and terminate
(C) It will not print anything and will not terminate
(D) It will print the binary representation of $n$ but will not terminate
36. Ans: (C)

Sol: Assume that input $\mathrm{n}=8$


It will be continued recursively and it will not print anything and not terminated.
37. A certain processor deploys a single - level cache. The cache block size is 8 words and the word size is 4 bytes. The memory system uses a $60-\mathrm{MHz}$ clock. To seryice a cache miss, the memory controller first takes 1 cycle to accept the starting address of the block, it then takes 3 cycles to fetch all the eight words of the block, and finally transmits the words of the requested block at the rate of 1 word per cycle. The maximum bandwidth for the memory system when the program running on the processor issues a series of read operations is $\qquad$ $\times 10^{6}$ bytes $/ \mathrm{sec}$.

## 37. Ans: 160

Sol: cache size $=8$ words
word size $=4 \mathrm{~B}$
cache size $=8 * 4 B=32 B$
Memory clock rate $=60-\mathrm{MHz}$
Memory cycle time $=\frac{1}{60 \mathrm{MHz}}=\frac{1}{60 * 10^{6}}$ seconds
No. of cycles needed to transfer 1 block ( 8 words)

$$
\begin{gathered}
\Rightarrow \quad \begin{array}{r}
1 \text { cycle for address } \\
+3 \text { cycles to fetch } 8 \text { words } \\
+8 * 1=8 \text { cycles to transmit }
\end{array} \\
\Rightarrow 12-\text { cycles }
\end{gathered}
$$

Time required to access and transfer 8 word (32B) from memory $=12 \times \frac{1}{60 \times 10^{6}}$ seconds

$$
=\frac{1}{5 \times 10^{6}} \text { seconds }
$$

In $\frac{1}{5 \times 10^{6}}$ second amount of data accessed=32bytes

$$
\begin{aligned}
\text { In } 1 \text { second amount of data accessed } & =\frac{32 \mathrm{~B}}{\frac{1}{5 \times 10^{6}} \text { seconds }} \\
& =32 \times 5 \times 10^{6} \text { Bytes } / \text { second } \\
& =160 \times 10^{6} \text { Bytes } / \text { second }
\end{aligned}
$$

38. There are $n$ unsorted arrays: $\mathrm{A}_{1}, \mathrm{~A}_{2}, \ldots ., \mathrm{A}_{n}$. Assume that $n$ is odd. Each of $\mathrm{A}_{1}, \mathrm{~A}_{2}, \ldots ., \mathrm{A}_{n}$ contains $n$ distinct elements. There are no common elements between any two arrays. The worst-case time complexity of computing the median of the medians of $\mathrm{A}_{1}, \mathrm{~A}_{2} \ldots \ldots, \mathrm{~A}_{\mathrm{n}}$ is
(A) $\mathrm{O}\left(\mathrm{n}^{2}\right)$
(B) $\Omega\left(n^{2} \log n\right)$
(C) $\mathrm{O}(\mathrm{n})$
(D) $O(n \log n)$
39. Ans: (A)
40. Consider that 15 machines need to be connected in a LAN using 8-port Ethernet switches. Assume that these switches do not have any separate uplink ports. The minimum number of switches needed is $\qquad$ .
41. Ans: 3

Sol: You lose one port on each for its network connection, so if you use 8 port switches, you have 7 available. Hence, to connect 15 devices, you would need 3.
40. Let G be any connected, weighted, undirected graph.
I. $G$ has a unique minimum spanning tree, if no two edges of $G$ have the same weight.
II. G has a unique minimum spanning tree, if, for every cut of $G$, there is a unique minimumweight edge crossing the cut.
Which of the above two statements is/are TRUE?
(A) Both I and II
(B) I only
(C) Neither I nor II
(D) II only
40. Ans: (A)
41. Consider the following four processes with arrival times (in milliseconds) and their length of CPU bursts (in milliseconds) as shown below:

| Process | P1 | P2 | P3 | P4 |
| :--- | :--- | :--- | :--- | :--- |
| Arrival time | 0 | 1 | 3 | 4 |
| CPU burst time | 3 | 1 | 3 | Z |

These processes are run on a single processor using preemptive Shortest Remaining Time First scheduling algorithm. If the average waiting time of the processes is 1 millisecond, then the value of Z is $\qquad$ .
41. Ans: 2

Sol: Gantt Chart:


At time $\mathrm{t}=4$
$P_{3}$ 's remaining Time $=3$
$\mathrm{P}_{4}$ 's remaining Time $=$ ' Z '
and avg waiting time $=\frac{2}{4}$
In order to make $P_{3}$ wait for 2 units of time, $Z$ should be taken as 2 , then the avg waiting time will become 1 .
$\therefore \mathrm{Z}$ should be 2
42. In an RSA cryptosystem, the value of the public modulus parameter $n$ is 3007 . If it is also known that $\varphi(\mathrm{n})=2880$, where $\varphi()$ denotes Euler's Totient Function, then the prime factor of $n$ which is greater than 50 is $\qquad$ .
42. Ans: 97

Sol: Given $\mathrm{n}=3007$
As per RSA p $=31$

$$
q=97
$$

$\mathrm{n}=\mathrm{p} \times \mathrm{q}=31 \times 97=3007$
Given $\varphi(\mathrm{n})=(\mathrm{p}-1)(\mathrm{q}-1)=30 \times 96=2880$
So Prime factor greater than 50 is 97
43. Consider the following sets:

S1. Set of all recursively enumerable languages over the alphabet $\{0,1\}$
S2. Set of all syntactically valid C programs
S3. Set of all languages over the alphabet $\{0,1\}$
S4. Set of all non-regular languages over the alphabet $\{0,1\}$
Which of the above sets are uncountable?
(A) S1 and S2
(B) S 1 and S 4
(C) S3 and S4
(D) S2 and S3
43. Ans: (C)

Sol: S1: Set of all RELs over $\{0,1\}$ is countable
S2: Set of all syntactically valid C program is countable
S3: Set of all languages over $\{0,1\}$ is uncountable
S4: Set of all non regular languages is uncountable
$\therefore \mathrm{S}_{3}$ and $\mathrm{S}_{4}$ are uncountable
44. Consider the following snapshot of a system running $n$ concurrent processes. Process $i$ is holding $\mathrm{X}_{\mathrm{i}}$ instances of a resource $\mathrm{R}, 1 \leq i \leq \mathrm{n}$. Assume that all instances of R are currently in use. Further, for all $i$, process $i$ can place a request for at most $\mathrm{Y}_{i}$ additional instances of R while holding the $\mathrm{X}_{i}$ instances it already has. Of the $n$ processes, there are exactly two processes p and q such that $\mathrm{Y}_{\mathrm{p}}=\mathrm{Y}_{\mathrm{q}}=0$. Which one of the following conditions guarantees that no other process apart from p and $q$ can complete execution?
(A) $\operatorname{Min}\left(\mathrm{X}_{\mathrm{p}}, \mathrm{X}_{\mathrm{q}}\right) \leq \operatorname{Max}\left\{\mathrm{Y}_{\mathrm{k}} \mid 1 \leq \mathrm{k} \leq \mathrm{n}, \mathrm{k} \neq \mathrm{p}, \mathrm{k} \neq \mathrm{q}\right\}$
(B) $\mathrm{X}_{\mathrm{p}}+\mathrm{X}_{\mathrm{q}}<\operatorname{Min}\left\{\mathrm{Y}_{\mathrm{k}} \mid 1 \leq \mathrm{k} \leq \mathrm{n}, \mathrm{k} \neq \mathrm{p}, \mathrm{k} \neq \mathrm{q}\right\}$
(C) $\operatorname{Min}\left(X_{p}, X_{q}\right) \geq \operatorname{Min}\left\{Y_{k} \mid 1 \leq k \leq n, k \neq p, k \neq q\right\}$
(D) $\mathrm{X}_{\mathrm{p}}+\mathrm{X}_{\mathrm{q}}<\operatorname{Max}\left\{\mathrm{Y}_{\mathrm{k}} \mid 1 \leq \mathrm{k} \leq \mathrm{n}, \mathrm{k} \neq \mathrm{p}, \mathrm{k} \neq \mathrm{q}\right\}$

## 44. Ans: (B)

Sol: - Two process p \& q are doing their work because they require zero more instances of resource.

- After Finishing they will release whatever they have i.e. $\left(X_{p}+X_{q}\right)$ and then other process can continue their execution.


## Holding Requesting

$$
\begin{aligned}
& \left.\mathrm{P}_{1}-\left(\begin{array}{l}
\mathrm{X}_{1} \\
\mathrm{P}_{2}-\mathrm{Y}_{1}=0 \\
\mathrm{X}_{2} \\
\mathrm{Y}_{3}-\mathrm{X}_{3}=0 \\
\mathrm{P}_{4}-\mathrm{X}_{4}-\mathrm{Y}_{3}=8 \\
\mathrm{Y}_{4}=7 \\
\mathrm{P}_{6}-\mathrm{X}_{5}-\mathrm{X}_{6}=15 \\
\mathrm{Y}_{6}=12
\end{array}\right\} \quad \begin{array}{l}
\text { Assume its } \mathrm{p} \\
\text { Lets take random value }
\end{array} \quad \begin{array}{l} 
\\
\end{array}\right\} \text { Assume its } \mathrm{C} \\
&
\end{aligned}
$$

Now, To guarantee that, any other process $\left(\mathrm{P}_{3}\right.$ to $\left.\mathrm{P}_{6}\right)$ will not complete execution, we must have less resources than the minimum request i.e. 7

So, if $\left(X_{1}+X_{2}\right)$ is less than 7 , no process will execute after $P_{1} \& P_{2}$.

$$
\left(\mathrm{X}_{1}+\mathrm{X}_{2}\right)<\operatorname{Min}\left(\mathrm{Y}_{3}, \mathrm{Y}_{4}, \mathrm{Y}_{5} \text { and } \mathrm{Y}_{6}\right)
$$

45. Let T be a full binary tree with 8 leaves. (A full binary tree has every level full). Suppose two leaves $a$ and $b$ of $T$ are chosen uniformly and independently at random. The expected value of the distance between a and b in T (i.e., the number of edges in the unique path between a and b ) is (rounded off to 2 decimal places) $\qquad$ .
46. Ans: 4.86

Sol: From a Full Binary Tree 'T' of 8 leaf nodes, Two leaf nodes, A and B are selected randomly and uniformly., then expected distance calculation is as follows:


Considering the above figure, there are 3 cases:
Case 1: The two leaf nodes are siblings, then Distance between siblings $(1,2)(3,4)(5,6)(7,8)=2$ Case 2: The two leaf nodes are NOT siblings, But their parents are siblings, then the Distance is $\{(1,3)(1,4)(5,7)(5,8)\}=4$

Case 3: The two leaf nodes are NOT siblings and their parents are also NOT siblings, but their grand parent is common, then Distance is $\{(1,5),(1,6),(1,7),(1,8)\}=6$
In general, The total number of ways of selecting two nodes from 8 leaves is $={ }^{8} \mathrm{C}_{2}=\frac{8 \times 7}{2 \times 1}=28$
Case 1 : Probability of 2 selected leaf nodes being siblings
$=$ number of favourable selections / total number of selections
So, $\frac{4}{{ }^{8} C_{2}}=\frac{4}{28}=\frac{1}{7}$

## Similarly for Case 2:

Favourable are $(1,3)(1,4)(2,3)(2,4)(5,7)(5,8)(6,7)(6,8)=8$
So, $\mathrm{P}=\frac{8}{{ }^{8} C_{2}}=\frac{8}{28}=\frac{2}{7}$

## Similarly for Case 3:

favourable $\Rightarrow \quad(1,5),(1,6)(1,7)(1,8)=4$

$$
\begin{aligned}
& (2,5),(2,6),(2,7),(2,8)=4 \\
& (3,5),(3,6),(3,7),(3,8)=4 \\
& (4,5),(4,6),(4,7),(4,8)=\underline{4}
\end{aligned}
$$

So, $\mathrm{P}=\frac{16}{{ }^{8} C_{2}}=\frac{16}{28}=\frac{4}{7}$
Hence, Case $1+$ Case $2+$ Case 3 , Probability is one as all the cases are considered

$$
\left\{\frac{1}{7}+\frac{2}{7}+\frac{4}{7}=\frac{1+2+4}{7}=\frac{7}{7}=1\right\}
$$

$$
\begin{aligned}
& \text { Expected distance }=\sum_{i=1}^{3} \text { probability } \times \text { distance } \\
& \frac{1}{7}(2)+\frac{2}{7}(4)+\frac{4}{7}(6)=\frac{2}{7}+\frac{8}{7}+\frac{2}{7}=\frac{34}{7}=4.86
\end{aligned}
$$

46. Consider the following $C$ program:
```
#include <stdio.h>
    int r()
    {
            static int num = 7;
    return num --;
    int main ()
    {
            for (r(); r(); r())
            printf ("%d", r ( )) ;
            return 0;
    }
```

Which one of the following values will be displayed on execution of the programs?
(A) 630
(B) 63
(C) 41
(D) 52
46. Ans: (D)

Sol: The for loop contain three parts and they are initialization, testing condition, updating values of the variable.

When we call $\mathrm{r}($ ), first time then it will return ' 7 '
When we call $r()$, second time then it will return 6 and ' 6 ' is non zero so the printf ("\%d", $r())$; will print the value ' 5 '.

When we call $\mathrm{r}($ ), from updating values then it will return 4,
When we call r() from testing condition then it will return 3 and it is non zero so $\operatorname{printf}(" \% \mathrm{~d}$ ", $\mathrm{r}(\mathrm{)})$ will print the value 2 so the output is 52 .
47. Consider the following C program:

```
#include <stdio.h>
int main ()
{
    int a[ ] ={2,4,6,8,10};
    inti, sum = 0, *b = a + 4;
    for (i= 0; i < 5; i + +)
    sum = sum + (*b-i)-*(b-i);
    printf ("%d\n", sum);
    return 0;
}
```

The output of the above C program is $\qquad$ .
47. Ans: 10

Sol:

48. Suppose Y is distributed uniformly in the open interval $(1,6)$. The probability that the polynomial $3 x^{2}+6 x Y+3 Y+6$ has only real roots is (rounded off to 1 decimal place) $\qquad$
48. Ans: 0.8

Sol: Probability density function of $Y=f(y)= \begin{cases}\frac{1}{6-1} & 1 \leq y \leq 6 \\ 0, & \text { otherwise }\end{cases}$
If the polynomial $3 x^{2}+6 x Y+3 Y+6$ has only real roots then
$36 \mathrm{Y}^{2}-36(\mathrm{Y}+2)>0$
$\Rightarrow\left(\mathrm{Y}^{2}-\mathrm{Y}-2\right)>0$
$\Rightarrow(\mathrm{Y}+1)(\mathrm{Y}-2)>0$
$\Rightarrow \mathrm{Y}>2$
Required probability $=\int_{5}^{6} \frac{1}{5} d y$

$$
=\frac{4}{5}=0.8
$$

49. Consider the following relations $\mathrm{P}(\mathrm{X}, \mathrm{Y}, \mathrm{Z}), \mathrm{Q}(\mathrm{X}, \mathrm{Y}, \mathrm{T})$ and $\mathrm{R}(\mathrm{Y}, \mathrm{V})$.

| $\mathbf{P}$ |  |  |
| :---: | :---: | :---: |
| $\mathbf{X}$ | $\mathbf{Y}$ | $\mathbf{Z}$ |
| X 1 | Y 1 | Z 1 |
| X 1 | Y 1 | Z 2 |
| X 2 | Y 2 | Z 2 |
| X 2 | Y 4 | Z 4 |


| $\mathbf{Q}$ |  |  |
| :---: | :---: | :---: |
| $\mathbf{X}$ | $\mathbf{Y}$ | $\mathbf{T}$ |
| X 2 | Y 1 | 2 |
| X 1 | Y 2 | 5 |
| X 1 | Y 1 | 6 |
| X 3 | Y 3 | 1 |


| $\mathbf{R}$ |  |
| :---: | :---: |
| $\mathbf{Y}$ | $\mathbf{V}$ |
| Y 1 | V 1 |
| Y 3 | V 2 |
| Y 2 | V 3 |
| Y 2 | V 2 |

How many tuples will be returned by the following relational algebra query?

$$
\prod_{X}\left(\sigma_{(P . Y=R . Y \wedge R . V=V 2)}(P \times R)\right)-\prod_{X}\left(\sigma_{(Q . Y=R . Y \wedge Q . T>2)}(Q \times R)\right)
$$

Answer: $\qquad$
49. Ans: 1

Sol:

| P |  |
| :---: | :---: |
| X Y Z | Y V |
| $\mathrm{X}_{1} \mathrm{Y}_{1} \mathrm{Z}_{1}$ | $\mathrm{Y}_{1} \mathrm{~V}_{1}$ |
| $\mathrm{X}_{1} \mathrm{Y}_{1} \mathrm{Z}_{2}$ | $\mathrm{Y}_{3} \mathrm{~V}_{2}$ |
| $\mathrm{X}_{2} \mathrm{Y}_{2} \mathrm{Z}_{2}$ | $\mathrm{Y}_{2} \mathrm{~V}_{3}$ |
| $\mathrm{X}_{2} \mathrm{Y}_{2} \mathrm{Z}_{4}$ | $\mathrm{Y}_{2} \mathrm{~V}_{2}$ |

Result of the expression $\prod_{X}\left(\sigma_{P, Y=R . Y \wedge R, V=V_{2}}{ }^{(P \times R)}\right)$ is $\frac{X}{X_{2}}$


Result of the expression $\prod_{x}\left(\sigma_{Q . Y=R . Y \wedge Q . T>2}{ }^{(Q \times R)}\right)$ is $\frac{X}{X_{1}}$
The result of $\left(\mathrm{X}_{2}\right)-\left(\mathrm{X}_{1}\right)=\mathrm{X}_{2}$
50. Consider the following C program:

## \# include <stdio.h>

int main ()
\{
float sum $=0.0, \mathrm{j}=1.0, \mathrm{i}=2.0$;
while ( $\mathrm{i} / \mathrm{j}>0.0625$ )
\{

$$
\mathrm{j}=\mathrm{j}+\mathrm{j} ;
$$

$$
\text { sum }=\operatorname{sum}+\mathrm{i} / \mathrm{j} ;
$$

printf("\%fln", sum);
\}
return 0;

The number of times the variable sum will be printed, when the above program is executed, is
$\qquad$ -
50. Ans: 5

Sol: Step 1: $\quad i=2.0, \quad j=1.0, \quad i / j>0.0625 \quad$ True
Step 2: $i=2.0, \quad j=2.0 \quad i / j>0.0625 \quad$ True
Step 3: $i=2.0, \quad j=4.0 \quad i / j>0.0625$ 。"True
Step 4: $i=2.0, \quad j=8.0 \quad i / j>0.0625 \quad$ True
Step 5: $i=2.0, \quad j=16.0 \quad i / j>0.0625 \quad$ True
Step 6: $\quad i=2.0, \quad j=32.0 \quad i / j>0.0625 \quad$ False

So the number of times $\operatorname{printf}(\% \% \mathrm{f} "$, sum) executed is 5 .
51. Consider the first order predicate formula $\varphi$ :

$$
\forall \mathrm{x}[(\forall \mathrm{z} \mathrm{z} \mid \mathrm{x} \Rightarrow((\mathrm{z}=\mathrm{x}) \vee(\mathrm{z}=1))) \Rightarrow \exists \mathrm{w}(\mathrm{w}>\mathrm{x}) \wedge(\forall \mathrm{z} \mid \mathrm{w} \Rightarrow((\mathrm{w}=\mathrm{z}) \vee(\mathrm{z}=1)))]
$$

Here 'a|b' denotes that 'a divides b ', where a and b are integers.
Consider the following sets:
S1. $\{1,2,3 . . . . .100\}$
S2. Set of all positive integers
S3. Set of all integers
Which of the above sets satisfy $\varphi$ ?
(A) S2 and S3
(B) $\mathrm{S} 1, \mathrm{~S} 2$ and S 3
(C) S1 and S2
(D) S1 and S3
51. Ans: (A)

Sol: Given predicate formula can be interpreted as:
"For each prime ' $x$ ', there exist prime ' $w$ ' where $w>x$ "
S1: $\{1,2,3, \ldots . .100\}$ fails to satisfy the formula because when $x=97$ there is no $w$ exist.
$\mathbf{S} 2$ and $\mathbf{S 3}$ are infinite sets, So for every x we can find w.
Therefore S2 \& S3 can satisfy the given formula.
52. Consider the following matrix:

$$
R=\left[\begin{array}{cccc}
1 & 2 & 4 & 8 \\
1 & 3 & 9 & 27 \\
1 & 4 & 16 & 64 \\
1 & 5 & 25 & 125
\end{array}\right]
$$

The absolute value of the product of Eigen values of R is $\qquad$ .
52. Ans: 12

Sol: $\mathrm{R}=\left[\begin{array}{cccc}1 & 2 & 4 & 8 \\ 1 & 3 & 9 & 27 \\ 1 & 4 & 16 & 64 \\ 1 & 5 & 25 & 125\end{array}\right]$ absolute value of product of eigen value of R is
Product of eigen values $=$ Det of R

$$
\begin{aligned}
& \text { Now }|R|=\left|\begin{array}{cccc}
1 & 2 & 4 & 8 \\
1 & 3 & 9 & 27 \\
1 & 4 & 16 & 64 \\
1 & 5 & 25 & 125
\end{array}\right| \\
& R_{2} \rightarrow R_{2}-R_{1}, R_{3} \rightarrow R_{3}-R_{1}, R_{4} \rightarrow R_{4}-R_{1}
\end{aligned}
$$

$$
=\left|\begin{array}{cccc}
1 & 2 & 4 & 8 \\
0 & 1 & 5 & 19 \\
0 & 2 & 12 & 56 \\
0 & 3 & 21 & 117
\end{array}\right|
$$

$$
|\mathrm{R}|=1 \times\left|\begin{array}{ccc}
1 & 5 & 19 \\
2 & 12 & 56 \\
3 & 21 & 117
\end{array}\right|
$$

$$
\mathrm{R}_{2} \rightarrow \mathrm{R}_{2}-2 \mathrm{R}_{1}, \mathrm{R}_{3} \rightarrow \mathrm{R}_{3}-3 \mathrm{R}_{1}
$$

$$
=1 \times\left|\begin{array}{ccc}
1 & 5 & 9 \\
0 & 2 & 18 \\
0 & 6 & 60
\end{array}\right|
$$

$$
=1 \times 1 \times(120-108)=12
$$

Now absolute value of product of Eigen value $=12$
53. A relational database contains two tables Student and Performance as shown below:

| Student |  | Performance |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Roll_no | Student_name | Roll_no | Subject_code | Marks |
| 1 | Amit | 1 | A | 86 |
| 2 | Priya | 1 | * B | 95 |
| 3 | Vinit | (1) | C | 90 |
| 4 | Rohan | - 2 | A | 89 |
| 5 | Smita | 2 | C | 92 |
|  |  | 3 | C | 80 |

The primary key of the Student table is Rool_no. For the Performance table, the columns Roll_no. and Subject_code together form the primary key. Consider the SQL query given below:

SELECT S.Student_name, sum(P.Marks)
FROM Student S, Performance P
WHERE P.Marks > 84
GROUP BY S.Student_name;

The number of rows returned by the above SQL query is $\qquad$ .

## 53. Ans: 5

Sol: As there is no join condition, it is a Cartesian Product of student and performance tables.
Because of the statement "Group by S.Student_name", the rows are divided into 5 groups, one for each student and for each group one row will be returned as output.
54. Let the set of functional dependencies $\mathrm{F}=\{\mathrm{QR} \rightarrow \mathrm{S}, \mathrm{R} \rightarrow \mathrm{P}, \mathrm{S} \rightarrow \mathrm{Q}\}$ hold on a relation schema $\mathrm{X}=(\mathrm{PQRS}) . \mathrm{X}$ is not in BCNF. Suppose X is decomposed into two schemas Y and Z , where $\mathrm{Y}=(\mathrm{PR})$ and $\mathrm{Z}=(\mathrm{QRS})$.
Consider the two statements given below:
I. Both Y and Z are in BCNF
II. Decomposition of X into Y and Z is dependency preserving and lossless

Which of the above statements is/are correct?
(A) II only
(B) Neither I nor II
(C) I only
(D) Both I and II
54. Ans: (A)

Sol: $\mathrm{F}=\{\mathrm{QR} \rightarrow \mathrm{S}, \mathrm{R} \rightarrow \mathrm{P}, \mathrm{S} \rightarrow \mathrm{Q}\}$
The decomposed relations $Y(P R)$ and $Z(Q R S)$ satisfying the dependencies $\{R \rightarrow P\}$ and $\{Q R \rightarrow S$, $\mathrm{S} \rightarrow \mathrm{Q}\}$ respectively.

Relation $Y$ is in BCNF but relation $Z$ is not in BCNF because in $S \rightarrow Q \mathrm{~S}$ is not a super key. All the dependencies of relation X is satisfying on relations Y and Z .
55. Assume that in a certain computer, the virtual addresses are 64 bits long and the physical addresses are 48 bits long. The memory is word addressible. The page size is 8 kB and the word size is 4 bytes. The Translation Look-aside Buffer (TLB) in the address translation path has 128 valid entries. At most how many distinct virtual addresses can be translated without any TLB miss?
(A) $4 \times 2^{20}$
(B) $16 \times 2^{10}$
(C) $256 \times 2^{10}$
(D) $8 \times 2^{20}$
55. Ans: (C)

Sol: As given memory is word addressable.
Word size $=4$ Bytes
Page size $=8 \mathrm{~KB}=\frac{8 \mathrm{~KB}}{4 \mathrm{~B}}$ words $=2 \mathrm{~K}$ words
Number of valid entries in TLB $=128$
128 entries can translate 128 page numbers in frame numbers.
Hence, number of distinct virtual addresses translated without TLB miss $=128 \times 2 \mathrm{~K}$ addresses

$$
=256 \times 2^{10}
$$

